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Tugas Pertemuan 12

1. Dapatkan transformasi Laplace untuk fungsi berikut dimana a, b, c, ω, dan θ adalah sebuah Konstanta.

j). $f(t) = \cos\frac{1}{2}t + 5\sin\frac{1}{2}t$

$$\mathcal{L}\{\cos\tfrac{1}{2}t + 5\sin\tfrac{1}{2}t\} = \mathcal{L}\{\cos\tfrac{1}{2}t\} + 5\,\mathcal{L}\{\sin\tfrac{1}{2}t\}$$

$$= \frac{s}{s^2+(\frac{1}{2})^2} + 5 \cdot \frac{1/2}{s^2+(\frac{1}{2})^2}$$

$$= \frac{s + 5/2}{s^2 + 1/4} \cdot \frac{4}{4}$$

$$= \frac{4s+10}{4s^2+1} \;//$$

Jawab $= \dfrac{4s+10}{4s^2+1}$ //

n) $f(t) = \sinh t + \cosh t$

$$\mathcal{L}\{\sinh t + \cosh t\} = \mathcal{L}\{\sinh t\} + \mathcal{L}\{\cosh t\}$$

$$= \frac{1}{s^2-1} + \frac{s}{s^2-1}$$

$$= \frac{\cancel{s+1}}{\cancel{(s+1)}(s-1)}$$

$$= \frac{1}{s-1} \;//$$

▶ Cara 2 : $\sinh t = \dfrac{e^t - e^{-t}}{2}$; $\cosh t = \dfrac{e^t + e^{-t}}{2}$; $\sinh t + \cosh t = \dfrac{e^t \cancel{- e^{-t}} + e^t \cancel{+ e^{-t}}}{2}$

$= e^t$

$\Rightarrow \mathcal{L}\{e^t\} = \dfrac{1}{s-1}$ //

Jawab $= \dfrac{1}{s-1}$ //

p). $f(t) = \begin{cases} 1, & 0 \le t \le 2 \\ -1, & t \ge 2 \end{cases}$ $\qquad \mathcal{L}\{f(t)\} = \begin{cases} 1/s, & 0 \le t \le 2 \\ -1/s, & t \ge 2 \end{cases}$

$$\bullet\!> \int_0^2 e^{-st}\,dt = \left[-\frac{1}{s}e^{-st}\right]_0^2$$

$$= -\frac{1}{s}e^{-s.2} - \left(-\frac{1}{s}e^0\right)$$

$$= \frac{1}{s} - \frac{1}{s}e^{-2s}$$

$$\bullet\!> \int_2^{+\infty} e^{-st}(-1)\,dt = \lim_{p\to+\infty} \int_2^p -e^{-st}\,dt$$

$$= \lim_{p\to+\infty} \left. -\frac{1}{-s}e^{-st}\right]_2^p$$

$$= \lim_{p\to+\infty} \frac{1}{s}e^{-ps} - \left(\frac{1}{s}e^{-2s}\right)$$

$$= \frac{1}{s}e^0 + \frac{1}{s}e^{-2s}$$

$$\int_0^{+\infty} e^{-st}[f(t)]\,dt = \int_0^2 e^{-st}[f(t)]\,dt + \int_2^{+\infty} e^{-st}[f(t)]\,dt$$

$$= \frac{1}{s} - \frac{1}{s}e^{-2s} - \frac{1}{s}e^{-2s} + \frac{1}{s}$$

$$= \frac{2-2e^{-2s}}{s} \;//$$

Jawab $= \dfrac{2(1-e^{-2s})}{s}$ //

2. Dapatkan invers transformasi Laplace untuk fungsi berikut di mana a, b, c, ω, dan θ adalah konstanta.

h). $F(s) = \dfrac{2s+1}{s^2-2s+2}$

$$\mathcal{L}^{-1}\left\{\frac{2s+1}{s^2-2s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s-1)+3}{(s-1)^2+1}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s-1)}{(s-1)^2+1}\right\} + \mathcal{L}^{-1}\left\{\frac{3}{(s-1)^2+1}\right\}$$

$$= 2.\mathcal{L}^{-1}\left\{\frac{(s-1)}{(s-1)^2+1}\right\} + \mathcal{L}^{-1}\left\{3.\frac{1}{(s-1)^2+1}\right\}$$

$$= 2.e^t\cos t + 3.e^t\sin t$$

Jawab = $2e^t\cos t + 3e^t\sin t$ //

j). $F(s) = \dfrac{2s-3}{s^2+2s+10}$

$$\mathcal{L}^{-1}\left\{\frac{2s-3}{s^2+2s+10}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s+1)-5}{(s+1)^2+3^2}\right\} = \mathcal{L}^{-1}\left\{\frac{2(s+1)}{(s+1)^2+3^2}\right\} - \mathcal{L}^{-1}\left\{\frac{5}{(s+1)^2+3^2}\right\}$$

$$= 2.\mathcal{L}^{-1}\left\{\frac{(s+1)}{(s+1)^2+3^2}\right\} - \mathcal{L}^{-1}\left\{\frac{5}{5}.\frac{3}{3}.\frac{5}{(s+1)^2+3^2}\right\}$$

$$= 2.e^{-t}\cos 3t - \frac{5}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+1)^2+3^2}\right\}$$

$$= 2e^{-t}\cos 3t - \frac{5}{3}.e^{-t}\sin 3t$$

Jawab = $2e^{-t}\cos 3t - \frac{5}{3}e^{-t}\sin 3t$ //

k). $F(s) = \dfrac{n\pi.L}{s^2+9}$

anggap $n\pi L$ = Konstanta.

$$\mathcal{L}^{-1}\left\{\frac{n\pi L}{s^2+9}\right\} = \frac{3}{3}.n\pi L\,\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\right\} = \frac{n\pi L}{3}\mathcal{L}^{-1}\left\{\frac{3}{s^2+3^2}\right\}$$

$$= \frac{\pi n L}{3}.\sin 3t$$ //

Jawab = $\dfrac{n\pi L}{3}\sin 3t$ //

3. f). $f(t) = e^t(a+bt)$

$= ae^t + bte^t$

$= \mathcal{L}\{ae^t + bte^t\}$

$= a\,\mathcal{L}\{e^t\} + b\,\mathcal{L}\{te^t\}$

$= a \cdot \dfrac{1}{s-1} + b \cdot \dfrac{1}{(s-1)^2}$ Jawab $= \dfrac{a}{s-1} + \dfrac{b}{(s-1)^2}$ //

$= \dfrac{a}{s-1} + \dfrac{B}{(s-1)^2}$

p). $f(t) = t \sinh 2t$

※ Sifat 5 : $\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$

$\mathcal{L}\{\underset{\uparrow \atop t}{t}\,\underbrace{\sinh 2t}_{f(t)}\} = -\dfrac{d}{ds}\left(\dfrac{2}{s^2-4}\right) = -\dfrac{0(s^2-4)-2(2s)}{(s^2-4)^2}$

$= -\dfrac{-4s}{(s^2-4)^2} = \dfrac{4s}{(s^2-4)^2}$

Jawab $= \dfrac{4s}{(s^2-4)^2}$ //

r). $f(t) = \dfrac{\cos at - \cos bt}{t}$

$= \mathcal{L}\left\{\dfrac{\cos at - \cos bt}{t}\right\} =$

Diketahui : $\mathcal{L}\{\cos at\} - \mathcal{L}\{\cos bt\}$

$= \dfrac{s}{s^2+a^2} - \dfrac{s}{s^2+b^2}$

Karena $\mathcal{L}\left\{\dfrac{f(t)}{t}\right\} = \int_s^{+\infty} f(u)\,du$ [Sifat 6]

$= \int_s^{+\infty}\left[\dfrac{u}{u^2+a^2} - \dfrac{u}{u^2+b^2}\right]du$

Misal : $p = u^2+a^2$ $\quad = \int_s^{+\infty} \dfrac{1}{2} \cdot \dfrac{2u\,du}{p} - \int_s^{+\infty} \dfrac{1}{2} \cdot \dfrac{2u\,du}{k}$

$dp = 2u\,du$

$k = u^2+b^2$ $\quad = \dfrac{1}{2}\left[\int_s^{+\infty} \dfrac{dp}{p} - \int_s^{+\infty} \dfrac{dk}{k}\right]$

$dk = 2u\,du$

$= \dfrac{1}{2}\left[\ln(u^2+a^2) - \ln(u^2+b^2)\right]_s^{+\infty}$

$= \dfrac{1}{2}\left[\ln \dfrac{u^2+a^2}{u^2+b^2}\right]_s^{+\infty}$

$$= \frac{1}{2} \ln \left[\frac{u^2+a^2}{u^2+b^2} \right]_s^{+\infty}$$

$$= \frac{1}{2} \ln \left[\frac{1+a^2/u^2}{1+b^2/u^2} \right]_s^{+\infty}$$

$$= \frac{1}{2} \ln \left[\frac{1+a^2/\infty}{1+b^2/\infty} - \frac{1+a^2/s^2}{1+b^2/s^2} \right]$$

$$= \frac{1}{2} \ln \left[1 - \frac{s^2+a^2}{s^2} \cdot \frac{s^2}{s^2+b^2} \right]$$

$$= \frac{1}{2} . \ln 1 - \frac{1}{2} \ln \left(\frac{s^2+a^2}{s^2+b^2} \right)$$

$$= -\frac{1}{2} \ln \left(\frac{s^2+a^2}{s^2+b^2} \right)$$

$$= \frac{1}{2} \ln \left(\frac{s^2+b^2}{s^2+a^2} \right) = F(s)$$

Jadi, $\mathcal{L}\left\{ \frac{\cos at - \cos bt}{t} \right\} = \frac{1}{2} \ln \left(\frac{s^2+b^2}{s^2+a^2} \right)$

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